before and after the shock wave; μ, λ_h, λ_e, D_A, coefficients of viscosity, atom—ion and electron thermal conductivities, and ambipolar diffusion; δ, surface emissivity; n_i, ion concentration; Pr, Prandtl number; Sc, Schmidt number; $l = \rho\mu/\rho_s\mu_s$, dimensionless parameter; h, e, subscripts referring to parameters of the atom—ion and the electron gas, respectively; ∞, s, w, subscripts referring to parameters of the gas in the incident flow, immediately behind the shock wave, and at the body, respectively.

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A CLASS OF MULTIPLE INTEGRALS OF TRANSFER THEORY

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We consider a class of multiple integrals of transfer theory under the assumption that the scattering field function may be exponential.

The scattering amplitude of two particles is determined by the Lippmann—Schwinger integral equation [1]

$$t(\mathbf{k}, \mathbf{k}', E) = V(\mathbf{k}, \mathbf{k}') + \int\limits_{\Omega_1} \frac{V(\mathbf{k}, \mathbf{p})\, t(\mathbf{p}, \mathbf{k}', E)}{E - p^2 + i0}\, d\mathbf{p}, \tag{1}$$

where

$$V(\mathbf{k}, \mathbf{k}') = \frac{1}{(2\pi)^3} \int\limits_{\Omega_2} \exp[-i(\mathbf{k} - \mathbf{k}')\mathbf{r}]\, V(\mathbf{r})\, d\mathbf{r};$$

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TABLE 1. Values of Integrals I_1 and I_2 as Functions of A for Various Parameters V_0, r_0, a, k, and k'

V_0	r_0	a	k	k'	I_1	I_2	A
0,36	2,46	0,2	0,3162	0,1	—0,881639	$-6 \cdot 10^{-9}$	6
				0,6	—0,688986	$1 \cdot 10^{-9}$	
				1,1	—0,350975	$-6 \cdot 10^{-10}$	
				1,6	—0,071328	$4 \cdot 10^{-10}$	
				2,1	0,045935	$-3 \cdot 10^{-10}$	
				2,6	0,038709	$2 \cdot 10^{-10}$	
				3,1	0,000272	$-2 \cdot 10^{-10}$	
				3,6	—0,014119	$1 \cdot 10^{-10}$	
			1,732	0,1	—0,018354	$-1 \cdot 10^{-10}$	6
				0,6	—0,038190	$3 \cdot 10^{-10}$	
				1,1	—0,066295	$-4 \cdot 10^{-11}$	
				1,6	—0,073715	$3 \cdot 10^{-11}$	
				2,1	—0,052481	$2 \cdot 10^{-11}$	
				2,6	—0,019986	$-4 \cdot 10^{-11}$	
				3,1	0,001905	$-5 \cdot 10^{-11}$	
				3,6	0,006459	$6 \cdot 10^{-11}$	
1,93	1,07	0,2	0,3162	0,1	—0,518142	$-2 \cdot 10^{-5}$	6
				0,6	—0,482343	$-9 \cdot 10^{-6}$	
				1,1	—0,404746	$4 \cdot 10^{-6}$	
				1,6	—0,304667	$2 \cdot 10^{-6}$	
				2,1	—0,203361	$-2 \cdot 10^{-6}$	
				2,6	—0,117112	$-5 \cdot 10^{-7}$	
				3,1	—0,054043	$2 \cdot 10^{-6}$	
				3,6	—0,014706	$-1 \cdot 10^{-7}$	
			1,732	0,1	—0,280559	$-1 \cdot 10^{-6}$	6
				0,6	—0,267689	$3 \cdot 10^{-7}$	
				1,1	—0,238249	$1 \cdot 10^{-7}$	
				1,6	—0,196781	$-4 \cdot 10^{-7}$	
				2,1	—0,149503	$-2 \cdot 10^{-7}$	
				2,6	—0,103019	$3 \cdot 10^{-7}$	
				3,1	—0,062809	$5 \cdot 10^{-8}$	
				3,6	—0,032126	$-3 \cdot 10^{-8}$	
62,8	0,2	0,3	0,3162	0,1	—2,703753	$-1 \cdot 10^{-4}$	12
				0,6	—2,524195	$-1 \cdot 10^{-5}$	
				1,1	—2,150211	$2 \cdot 10^{-5}$	
				1,6	—1,695228	$-1 \cdot 10^{-5}$	
				2,1	—1,258822	$3 \cdot 10^{-6}$	
				2,6	—0,895332	$3 \cdot 10^{-6}$	
				3,1	—0,618634	$-5 \cdot 10^{-6}$	
				3,6	—0,419874	$4 \cdot 10^{-6}$	
				4,1	—0,282252	$3 \cdot 10^{-6}$	
				4,6	—0,189138	$-2 \cdot 10^{-6}$	
			1,732	0,1	—1,588219	$-2 \cdot 10^{-5}$	12
				0,6	—1,537386	$-2 \cdot 10^{-6}$	
				1,1	—1,417504	$-2 \cdot 10^{-6}$	
				1,6	—1,240264	$-2 \cdot 10^{-6}$	
				2,1	—1,027643	$1 \cdot 10^{-7}$	
				2,6	—0,807986	$3 \cdot 10^{-7}$	
				3,1	—0,606800	$-5 \cdot 10^{-7}$	
				3,6	—0,439390	$-3 \cdot 10^{-7}$	
				4,1	—0,309877	$2 \cdot 10^{-7}$	
				4,6	—0,214822	$1 \cdot 10^{-8}$	

V(**r**) is a spherically symmetric function of **r** which can be expanded in a series of Legendre polynomials

$$\frac{1}{E - p^2 \pm i0} = P\left(\frac{1}{E - p^2}\right) \mp i\pi\delta(E - p^2), \; E > 0; \tag{2}$$

P is a symbol for the principal value of the integral, and $\delta(r)$ is the Dirac delta function.

It is known that the function t(**k**, **k**'; E) can be expanded in Legendre polynomials for all real values of k and k'. This expansion reduces the three-dimensional equation (1) to a one-dimensional equation of the form [2]

$$t_l(k, k', E) = V_l(k, k') + \frac{2}{\pi}\int\limits_0^\infty \frac{V_l(k, p)\, t_l(p, k', E)\, p^2}{E - p^2 + i0}\, dp, \tag{3}$$

where

$$V_l(k, k') = \int\limits_0^\infty j_l(k, r) V(r) j_l(k', r) r^2 dr. \tag{4}$$

For a square well potential $V(r) = -V_0$ for $r < R_0$ and $V(r) = 0$ for $r > R_0$ the integral in (3) can be evaluated in terms of Bessel functions. For a potential of another form it is expedient to evaluate this integral numerically, e.g., by successive approximations or by the Padé method [2]. In both cases it is necessary to evaluate terms of the series

$$t_l(k, k', E) = V_l(k, k') + \frac{2}{\pi}\int_0^\infty \frac{V_l(k, p)V_l(p, k')p^2}{(E-p^2+i0)}dp$$

$$+\left(\frac{2}{\pi}\right)^2\int_0^\infty\int_0^\infty \frac{V_l(k, p)V_l(p, p')V_l(p', k')p^2(p')^2}{(E-p^2+i0)(E-(p')^2+i0)}dpdp'$$

$$+\left(\frac{2}{\pi}\right)^3\int_0^\infty\int_0^\infty\int_0^\infty \frac{V_l(k, p)V_l(p, p')V_l(p', p'')V_l(p'', k')p^2(p')^2(p'')^2}{(E-p^2+i0)(E-(p')^2+i0)(E-(p'')^2+i0)}dpdp'dp''+\dots \quad (5)$$

We consider the integrals in (5) for a function V(r) of the form

$$V(r) = -V_0[c+\exp((r-r_0)/a)]^{-1};\ c = 0,\ -1,\ 1. \quad (6)$$

With this choice of V(r) these integrals can be evaluated numerically after they are reduced to integrals with finite limits. The choice of the upper limits in Eqs. (4) and (5) to ensure the required accuracy of the calculation depends on the values of the fixed parameters of the potential V(r) and the variables k, k', and E.

We consider the integrals appearing in the amplitude $t_0(k, k'; E)$ when $0 < k, k' < 4\ F^{-1}$, $0 < E < 16\ F^{-2}$.

For $l = 0$ the integral (4) has the form

$$V_0(k, k') = \int_0^\infty \frac{\sin(kr)V(r)\sin(k'r)}{kk'}dr. \quad (7)$$

For a potential of the form (6) $V_0(k, k')$ is

$$V_0(k, k') = -V_0\int_0^\infty \frac{\sin(kr)\sin(k'r)}{kk'[\exp((r-r_0)/a)+c]}dr. \quad (8)$$

We write (8) as the sum of two integrals

$$V_0(k, k') = I_1 + I_2,$$

where

$$I_1 = -V_0\int_0^A \frac{\sin(kr)\sin(k'r)}{kk'(c+\exp((r-r_0)/a))}dr,$$

$$I_2 = -V_0\int_A^\infty \frac{\sin(kr)\sin(k'r)}{kk'(c+\exp((r-r_0)/a))}dr. \quad (9)$$

Point A is chosen so that the error made in neglecting I_2 is no more than 10^{-8}. The values of k and k' do not affect the position of point A substantially. The value of (8) is determined by the function $\exp[(r-r_0)/a]$ in the denominator. It is estimated that for $A \le 21a + r_0$ the error made by neglecting I_2 in (9) is no more than 10^{-8}. This estimate for the integral (7) ($l = 0$) is valid also for $l \neq 0$, but is somewhat of an overestimate.

Thus, the convergence of (4) for a potential well with a diffuse edge depends on the diffuseness parameter and the radius of the well.

Using (2) we rewrite series (5) in the form [3]

$$t(k, k', E) = V(k, k') + \frac{2}{\pi}\left\{\int_0^\infty \frac{[V(k, p)V(p, k')p^2 - V(k, k)V(k, k')E]}{(E-p^2)}dp\right.$$

$$\left. - i\frac{\pi V(k, k)V(k, k')E^{1/2}}{2}\right\} + \frac{4}{\pi^2}$$

$$\times\left\{\int_0^\infty\int_0^\infty \frac{[V(k,p)\,V(p,p')\,V(p',k')p^2(p')^2 - V^2(k,k)\,V(k,k')\,E^2]}{(E-p^2)(E-(p')^2)}\,dp\,dp' - \frac{\pi^2}{4}EV^2(k,k)\,V(k,k')\right\} + \ldots . \quad (10)$$

The upper limit in integrals of the form (10) was estimated by a numerical experiment. For various fixed values of E, k, and k' within the range of their variation the integral over p was decomposed into a sum of integrals and the series was broken off when the integrand was smaller than 10^{-4} in absolute value. The calculation was performed by using the approximate Gaussian formula.

Table 1 lists the results of numerical calculations of I_1 and I_2 for various choices of the parameters of the Woods–Saxon potential.

It is clear from Table 1 that the upper limit A of the integrals in (9) is most sensitive to the parameter $|V_0|$, the depth of the potential well. An accuracy of 10^{-4} in the calculations will certainly be ensured if the value of A is taken an order of magnitude larger than $|V_0|$. The remaining parameters will be arbitrary within the indicated limits.

NOTATION

k, k', wave vectors; E, energy of system; $i = \sqrt{-1}$; r, radius-vector; Ω_1, range of variation of wave vector; Ω_2, range of variation of radius-vector; V_0, depth of potential well; a, diffuseness parameter; r_0, radius of potential well; 1 F = 10^{-13} cm.

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